The Distribution of Willingness-To-Pay Tolls for Time Savings

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Several important issues regarding toll-roads, HOT lanes and transportation improvements depend on the Willingness-to-Pay tolls for travel time saving. In my limited experience, estimates of the WTP mostly treat it as a constant for any given population, dependent perhaps on average income.

Of course in any real travel population, values placed on — and willingness to pay for — travel time savings (WTP) differ widely from individual to individual and depending on circumstances of any particular trip. For some purposes, e.g. deciding optimum or maximal toll levels, it appears necessary to treat WTP as a random variate among a given traveler population described statistically in terms of its statistical distribution, the fraction of travelers willing to pay toll x.

The recent publication of experience on the SR-91 tollway in Orange and Riverside counties, California[1] provides a useful opportunity to estimate that distribution function. Figure 1 below [from [1]] plots the percent of total freeway traffic using the toll lanes along with the time saving as functions of time-of-day.

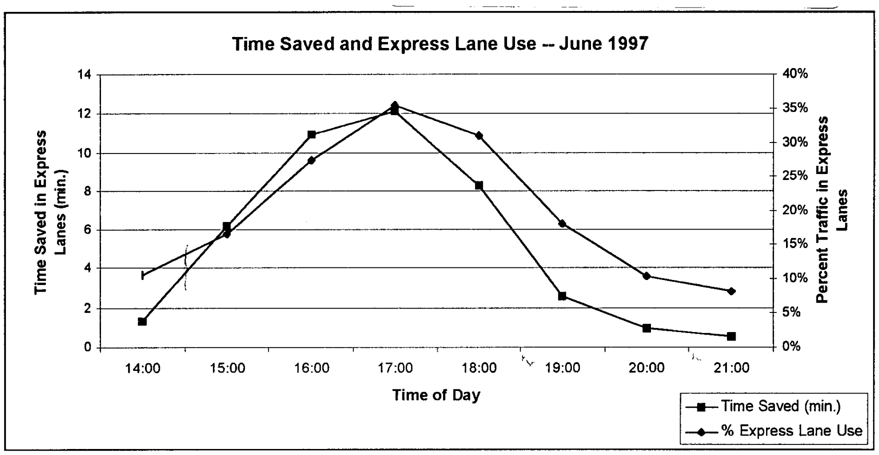


Figure 1. Time saved and Percent of traffic using the toll lane by Time of Day during PM traffic peak on the SR-91.

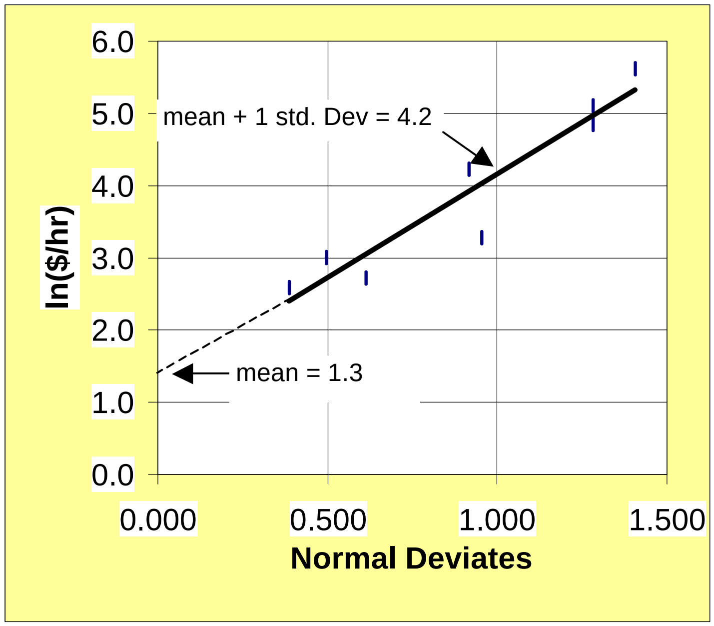
During this entire period the toll was $2.75. The data read from this curve are given in Table 1 along with the implied specific toll, S, $/hour, ranging from $13.31 to $275 per hour.

[1] Edward Sullivan, PM, "Evaluating the Impacts of the SR-91 Variable-Toll Express lane Facility", Cal Poly University, San Luis Obisbo, May 1998. On-line at http://ceenve.calpoly.edu/sullivan/sr91/sr91.htm

Having reason to suspect a log-normal distribution, in Figure 2 we plot y=ln(S), the natural logarithm of S, versus x, the standard normal deviate (or probits) corresponding to the fraction of time, F. That is, the quantity x is defined as the inverse cumulative normal probability function of F, implicitly by:

$$F = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp(-\frac{t^2}{2})\, dt$$

Figure 2. Ln($/hr) vs. x(F) .



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The straight line is the regression fit to the data and is a reasonably good fit, suggesting that the log-normal is as good a fit as any at least within the range of the data, S=13.35 to 275 $/hr. From the regression intercepts at 0.0 and 1.0 we read the values of mean, $m_{y,}$ and mean + 1 standard deviation, s_y of ln(S)

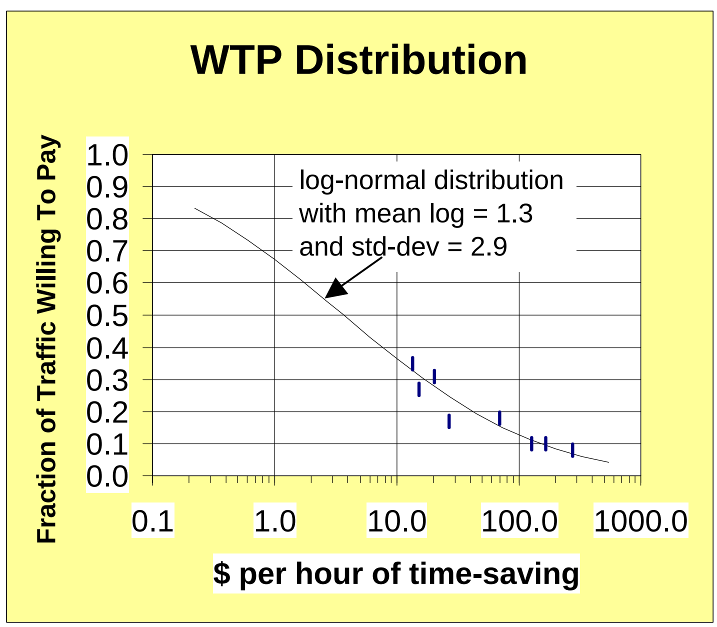
$m_y = 1.3$

and

$s_Y = 2.9$.

Finally, in table 1 we tabulate values of the cumulative distribution function, and plot it versus S in Figure 3:

Figure 3. Willingness-to-Pay for Time Savings



Note that the mean. 1.3 and standard deviation, 2.9 here are those of y=ln(S) and not of S itself.

We can derive the latter, however, from the definition of the log-normal distribution. In general, if y = ln(x/c) and y is normally distributed with mean, $m_{y,}$ and standard deviation, $s_{y,}$ then the mean and standard deviation of the x distribution are given by

$$m_x = c \exp\left\{\frac{s_y^2}{2} + m_y\right\}$$

and

$$s_x = c\left[\exp\left(2s_y^2 + 2m_y\right) - \exp\left(s_y^2 + 2m_y\right)\right]^{\frac{1}{2}}$$

In the present case with c = 1($/hr), m_y=1.3 and s_y = 2.9, we find

m_x = 246 $/hr
s_x = 16,479 $/hr

These surprisingly large results were confirmed by simple spreadsheet simulation. They are a result of the fact that the high-side tails of a log-normal distribution fall off VERY slowly as compared to a normal distribution. In this case, the mean of the distribution lies at about the upper 92 %ile rather than near the median as is the case in near normal distributions.

Clearly both results depend on values of the distribution well beyond the limit of validity of the data of about $275 /hr so the specific estimates of mean and standard deviation should be regarded only as placeholders for a better, determination based on a much more extensive data set. Nevertheless, the qualitative message may be quite significant. It is that average willingness to pay, WTP, in the strict statistical sense may be much greater than the median WTP or any reasonable estimate of VOT. It also means that maximal revenue production tolls during off-peak hours may be far greater than would seem to make sense from a Value-Of-Time perspective; in other words that there may be a significant disconnect between WTP and VOT for small time-savings. One may speculate that for some small fraction of drivers, as long as the absolute toll is below some threshold, the time saving is irrelevant and they pay the toll with little or no thought to $/hr cost-effectiveness.

These results also illustrate that any single point value of WTP tolls for time saving is liable to be quite misleading, depending on the application and suggest that more effort be directed to determining its statistical distribution properties.

Table 1. SR-91 Fraction of Traffic / Time saving (Ref 1)

Time Saved	% Traffic Using, F	Toll	Specific toll	ln(Specific Toll)	Normal Deviate, x(F)
min	%	$	$/hr	ln($/hr)	Probits
1.3	10%	2.75	126.92	4.8	1.282
6.2	17%	2.75	26.61	3.3	0.954
10.9	27%	2.75	15.14	2.7	0.613
12.4	35%	2.75	13.31	2.6	0.385
8.2	31%	2.75	20.12	3.0	0.496
2.4	18%	2.75	68.75	4.2	0.915
1	10%	2.75	165.00	5.1	1.282
0.6	8%	2.75	275.00	5.6	1.405